

Form factors for $B \longrightarrow \pi l \nu$ decay in a model constrained by chiral symmetry and quark model

Riazuddin *

*Department of Physics, King Fahd University of Petroleum and Minerals,
Dhahran 31261, Saudi Arabia
and National Center for Physics, Quaid-e-Azam University, Islamabad 45320,
Pakistan.*

T.A. Al-Aithan †

*Department of Physics, King Fahd University of Petroleum and Minerals,
Dhahran 31261, Saudi Arabia.*

Amjad Hussain Shah Gilani ‡

*Department of Physics and National Center for Physics, Quaid-e-Azam
University, Islamabad 45320, Pakistan.
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The form factors for the $B \longrightarrow \pi$ transition are evaluated in the entire momentum transfer range by using the constraints obtained in the framework combining the heavy quark expansion and chiral symmetry for light quarks and the quark model. In particular, we calculate the valence quark contributions and show that it together with the equal time commutator contribution simulate a B -meson pole q^2 -dependence of form factors in addition to the usual vector meson B^* -pole diagram for $B \longrightarrow \pi l \nu$ in the above framework. We discuss the predictions in our model, which provide an estimate of $|V_{ub}|^2$. PACS number(s): 13.20.-V, 12.39.Ki

I. INTRODUCTION

There has been great interest in the study of semileptonic decays of heavy mesons as they provide a testing ground for heavy-quark effective theory (HQET). The symmetry underlying this theory allows to derive model independent predictions for the form factors near the zero recoil point. On the other hand for heavy -to-light semileptonic transitions, there exists no symmetry principle to guide us. However, here the heavy quark expansion can be combined with chiral symmetry and PCAC for final pseudoscalar mesons [1–5]. We focus on $B \longrightarrow \pi l \nu$ which is important for the evaluation of the CKM matrix element V_{ub} .

There have been many calculations [1–26] for $B \longrightarrow \pi l \nu$ form factors, using different approaches, in the past. In this paper we follow the approach used in ref. 4. In $B \longrightarrow \pi l \nu$ decay, the vector meson pole B^* , with mass degenerate with B in the heavy quark symmetry limit, dominates the transition amplitude at the zero recoil point. We calculate the valence quark contribution, and find that it together with the equal-time contribution is as important as the B^* -meson pole dominance of the form factors since the former two simulate a B -meson pole like q^2 dependence of the form factors. We perform this calculation in a framework compatible with chiral symmetry and eliminate the B -meson bound state function in favor of B -meson decay constant f_B which can likewise be calculated in the valence quark approximation. This procedure gives us form factors for q^2 in the range determined by $E_\pi < \Lambda$ (~ 1 GeV), where Λ is some interaction scale, below which chiral symmetry should be valid. This constraint is then built into an extrapolation function for $f_+(q^2)$ which determines $f_+(q^2)$ in the entire q^2 -range. Finally, we compare our results with some of the earlier calculations and also obtain an estimate of $|V_{ub}|$ by using CLEO data.

II. CURRENT ALGEBRA CONSTRAINTS

The relevant hadronic matrix elements for the $B \rightarrow \pi l \nu$ decay is defined as

*E-mail address: riazuddins@yahoo.com

†E-mail address: alaithan@kfupm.edu.sa

‡E-mail address: ahgilani@yahoo.com

$$\begin{aligned}
T_\mu &= \langle \pi^+(k) | (V_\mu) | B^0(P) \rangle \\
&= f_+(t)(P+k)_\mu + f_-(t)q_\mu
\end{aligned} \tag{1}$$

with V_μ the weak vector current $V_\mu = \bar{u}\gamma_\mu b$ and $q = p - k, t = q^2$. Following Ref. 4, the current algebra constraint at $k^2 = 0$ (but not $k = 0$) is

$$\tilde{T}_\mu = -\left(\frac{ik^\lambda}{f_\pi}M_{\lambda\mu}^{B^*} + T_\mu^{B^*}\right) + \frac{f_B}{f_\pi}p_\mu - i\frac{k^\lambda}{f_\pi}\widetilde{M}_{\lambda\mu} \tag{2}$$

where the tilde denotes that B^* pole terms have been separated out from T_μ and from $M_{\lambda\mu}$ respectively, and $f_\pi = 0.132\text{GeV}$. The third term on the right hand side of Eq.(2) comes from the equal time commutator. $M_{\lambda\mu}$ is defined as

$$M_{\lambda\mu} = i \int d^4x \exp(ik \cdot x) \langle 0 | T(A_\lambda^{1-i}(x), V_\mu(0)) | B^0(P) \rangle \tag{3}$$

where A_λ^{1-i2} is the axial vector current $\bar{d}\gamma_\lambda\gamma_5 u$. The term within the parenthesis in Eq. (2) is easily evaluated to be $-g_{BB\pi}f_{B^*}/m_B^2$ and one obtains

$$\begin{aligned}
T_\mu &= T_\mu^{B^*} + \tilde{T}_\mu \\
&= \frac{g_{B^*B\pi}f_{B^*}}{M_{B^*}^2 - t} \left[(P+k)_\mu - \frac{M_B^2 - m_\pi^2}{M_{B^*}^2} q_\mu \right] - \frac{g_{B^*B\pi}f_{B^*}}{M_{B^*}^2} q_\mu + \frac{f_B}{f_\pi} \frac{1}{2} [(P+k)_\mu + q_\mu] - \frac{ik^\lambda}{f_\pi} \widetilde{M}_{\lambda\mu}
\end{aligned} \tag{4}$$

where one can identify $\widetilde{M}_{\lambda\mu}$ as a source of corrections to the leading B^* pole contribution. Next we identify such corrections, as valence quark contribution to $\widetilde{M}_{\lambda\mu}$ which we evaluate in the next section.

III. VALENCE QUARK CONTRIBUTION

The valence quark contribution is shown in Figure 1. Its evaluation gives

$$\widetilde{M}_{\lambda\mu} = i \int \frac{d^3K}{(2\pi)^3} A_{\lambda\mu} \tag{5}$$

where $A_{\lambda\mu}$ is the matrix element of Figure 1:

$$\begin{aligned}
A_{\lambda\mu} &= -i \left(\sqrt{2M_B} \frac{1}{\sqrt{2}} \sqrt{3} \phi_B(\mathbf{K}) \bar{u}^i(p_b) (\gamma_5)_i^j v(p_d)_j \right) \sqrt{\frac{m_d}{p_{d0}}} \bar{v}^r(p_d) (\gamma_\lambda \gamma_5)_r^s \frac{(m_u + \not{p}_u)_s^m}{p_u^2 - m_u^2} (\gamma_\mu)_m^n u_n(p_b) \sqrt{\frac{m_b}{p_{b0}}}
\end{aligned} \tag{6}$$

In equation (6), the term within the parenthesis is the bound state wave function of B-meson, $\sqrt{3}$ being the color factor. We define the variables $\mathbf{K} = \mathbf{p}_b - \mathbf{p}_d$, $\mathbf{P} = \mathbf{p}_b + \mathbf{p}_d$, so that \mathbf{K} is the relative momentum and \mathbf{P} is the centre of mass momentum of the $b\bar{d}$ system. Eq.(6) gives

$$\begin{aligned}
A_{\lambda\mu} &= -i\sqrt{2m_B} \frac{1}{\sqrt{2}} \sqrt{3} \sqrt{\frac{m_d m_b}{p_{d0} p_{b0}}} \frac{1}{4m_b m_d} \phi_B(\mathbf{K}) \frac{1}{p_u^2 - m_u^2} \\
&\quad \times \{ -Tr(\gamma_\lambda \gamma_5) (m_u + \not{p}_u) \gamma_\mu (m_b + \not{p}_b) \gamma_5 (\not{p}_d - m_d) \}
\end{aligned} \tag{7}$$

The above $-Tr$ is evaluated to be $(m_u = m_d = m_q)$

$$4 \{ g_{\lambda\mu} (m_q^2 m_b - m_q p_u \cdot p_b + m_q p_b \cdot p_d - m_b p_u \cdot p_d) - m_b (p_{u\mu} p_{d\lambda} + p_{u\lambda} p_{d\mu}) - m_q p_{b\lambda} (p_u - p_d)_\mu - m_q p_{b\mu} (p_u + p_d)_\lambda \} \tag{8}$$

Working in the rest frame of B-meson ($\mathbf{P} = 0$) where

$$\begin{aligned}
\left(\frac{\mathbf{K}^2}{4} + m_b^2 \right)^{1/2} &= \frac{M_B^2 + (m_b^2 - m_q^2)}{2M_B}, \\
K_0 &= \frac{m_b^2 - m_q^2}{2M_B}.
\end{aligned}$$

and

$$p_u^2 - m_u^2 = \frac{M_B^2 - m_b^2 + m_q^2}{2} \left(1 - \frac{q^2}{M_B^2} \right) - \mathbf{q} \cdot \mathbf{K} \quad (9)$$

$$-ik^\lambda A_{\lambda\mu} = 4C(\mathbf{K}) \frac{-1}{L' + \mathbf{q} \cdot \mathbf{K}} \{ (a + b\mathbf{q} \cdot \mathbf{K})k_\mu + (a' - b\mathbf{q} \cdot \mathbf{K})(K - q)_\mu \} \quad (10)$$

Here

$$C(\mathbf{K}) = \sqrt{2M_B} \frac{1}{\sqrt{2}} \sqrt{3} \sqrt{\frac{m_d m_b}{p_{d0} p_{b0}}} \frac{1}{4m_b m_d} \phi_B(K), \quad (11)$$

$$L' = -\frac{M_B^2 - m_b^2 + m_q^2}{2} \left(1 - \frac{q^2}{M_B^2} \right) \quad (12)$$

$$a = \frac{M_B^2 - (m_b - m_q)^2}{2} \left\{ -\frac{1}{2}(m_b + m_q) \left(1 - \frac{q^2}{M_B^2} \right) + 2m_q \right\} \quad (13)$$

$$b = \frac{1}{2}(m_b - m_q) \quad (14)$$

$$a' = \frac{1}{4} (M_B^2 - (m_b - m_q)^2) (m_b + m_q) \left(1 - \frac{q^2}{M_B^2} \right) \quad (15)$$

When Eq.(10) is put in (5) and the angular integration is carried out, one gets for example,

$$2\pi \int_{-1}^1 \frac{a + b\mathbf{q} \cdot \mathbf{K}}{L' + \mathbf{q} \cdot \mathbf{K}} = 2\pi \left\{ \frac{a - bL'}{|\mathbf{q}||\mathbf{K}|} \ln \frac{L' + |\mathbf{q}||\mathbf{K}|}{L' - |\mathbf{q}||\mathbf{K}|} + 2b \right\} \quad (16)$$

Then, noting that if $\phi_B(\mathbf{K})$ is of Gaussian type, $|\mathbf{K}| \simeq 0$ dominates in the K -integration, so that we can expand the logarithm in Eq.(16) and thus this equation reduces to

$$2\pi \left\{ \frac{a - bL'}{|\mathbf{q}||\mathbf{K}|} \frac{2|\mathbf{q}||\mathbf{K}|}{L'} + 2b \right\} = 4\pi \frac{a}{L'} \quad (17)$$

Further $4\pi \int K^2 dK \phi_B(K)$ becomes $\int d^3K \phi_B(K)$, which is the Fourier transform of the wave function at the origin which we write as $\phi_B(0)$. As far as the intergration involving K_μ is concerned, it is easy to see that the angular integration involving \mathbf{K} gives zero while that over K_0 gives $4\pi K_0 = 4\pi \frac{m_b^2 - m_q^2}{M_B^2} M_B$ so that in the rest frame of B-meson the angular integration involving K_μ gives $4\pi \frac{m_b^2 - m_q^2}{M_B^2} P_\mu$. Thus finally we obtain

$$-ik^\lambda \widetilde{M}_{\lambda\mu} = -4C(0) \left[\frac{a}{L'} k_\mu + \frac{a'}{L'} \left(\frac{m_b^2 - m_q^2}{M_B^2} P_\mu - q_\mu \right) \right] \quad (18)$$

where

$$C(0) = \sqrt{2M_B} \frac{1}{\sqrt{2}} \sqrt{3} \frac{1}{4m_b m_q} \phi_B(0) \quad (19)$$

Thus, writing $k_\mu = \left(\frac{P+k}{2} \right)_\mu - \frac{1}{2}q_\mu$, $P_\mu = \left(\frac{P+k}{2} \right)_\mu + \frac{1}{2}q_\mu$, we finally obtain the valence quark contribution to the form factors f_\pm as

$$f_+^{\text{valence}}(q^2) = -\frac{4C(0)}{2f_\pi L'} \left\{ a + \left(\frac{m_b^2 - m_q^2}{M_B^2} \right) a' \right\} \quad (20)$$

$$f_-^{\text{valence}}(q^2) = \frac{4C(0)}{2f_\pi L'} \left\{ a + \left(\frac{2M_B^2 - m_b^2 + m_q^2}{M_B^2} \right) a' \right\} \quad (21)$$

To eliminate $4C(0)$, we consider the matrix elements

$$\langle 0 | A_\lambda | B(p) \rangle = i f_B p_\lambda \quad (22)$$

which, when evaluated in the same valence quark approximation employed for the calculation of $-ik_{\lambda\mu}^\lambda \widetilde{M}_{\lambda\mu}$, give

$$f_B = \frac{4C(0)}{2M_B^2} (m_b + m_q) [M_B^2 - (m_b - m_q)^2] \quad (23)$$

Then, using Eqs.(11–13, 15) and (23), we obtain from Eq.(20, 21)

$$f_+^{\text{valence}}(q^2) = \frac{f_B}{2f_\pi} \left\{ -1 + \frac{4m_q M_B^2}{(m_b + m_q)(M_B^2 - (m_b - m_q)^2)} \frac{1}{1 - q^2/M_B^2} \right\} \quad (24)$$

$$f_-^{\text{valence}}(q^2) = -\frac{f_B}{2f_\pi} \left\{ 1 + \frac{4m_q M_B^2}{(m_b + m_q)(M_B^2 - (m_b - m_q)^2)} \frac{1}{1 - q^2/M_B^2} \right\} \quad (25)$$

IV. COMBINED CONTRIBUTION TO $F_+(Q^2)$ IN THE CHIRAL SYMMETRY LIMIT

Using Eq. (4) $[t = q^2]$, we obtain $[f_{B^*} = M_B f_B]$ [4]

$$f_+(t) = \frac{f_B}{2f_\pi} + f_+^{\text{valence}}(t) + \frac{g_{B^*B\pi} f_B M_B}{M_{B^*}^2 - t} \quad (26)$$

$$f_-(t) = \frac{f_B}{2f_\pi} + f_-^{\text{valence}}(t) + \frac{g_{B^*B\pi} f_B M_B}{M_{B^*}^2} - \frac{M_B^2 - m_\pi^2}{M_{B^*}^2} \frac{g_{B^*B\pi} f_B M_B}{M_{B^*}^2 - t} \quad (27)$$

The coupling constant $g_{B^*B\pi}$ has been parametrized as

$$g_{B^*B\pi} = \frac{\lambda M_B}{f_\pi} \quad (28)$$

where λ lies in the range $0.3 \leq \lambda \leq 0.7$.

We combine the equal time contribution $\frac{f_B}{2f_\pi}$ with $f_\pm^{\text{valence}}(q^2)$ and call it continuum contribution:

$$f_\pm^{\text{cont.}}(q^2) = \pm \lambda_c \frac{f_B}{2f_\pi} \frac{1}{1 - q^2/M_B^2} \quad (29)$$

while B^* -pole contributions, using the parametrization (28) $M_{B^*} \simeq M_B$, $m_\pi^2 = 0$, are

$$f_+^{B^*}(q^2) = \lambda \frac{f_B}{f_\pi} \frac{1}{1 - q^2/M_B^2} \quad (30)$$

$$f_-^{B^*}(q^2) = -\lambda \frac{f_B}{f_\pi} \left\{ 1 + \frac{1}{1 - q^2/M_B^2} \right\} \quad (31)$$

In Eq.(29),

$$\lambda_c = \frac{4m_q/m_b}{\left(1 + \frac{m_q}{m_b}\right) \left[1 - \frac{m_b^2}{M_B^2} \left(1 - \frac{m_q}{m_b}\right)^2\right]} \quad (32)$$

We wish to point out that the continuum contribution to the form factors consisting of equal time commutator and valence quark contributions simulate a B -meson pole q^2 -dependence of form factors $f_\pm(q^2)$. This seems to follow a general result in quark annihilation model [27] of decay of a pseudoscalar meson into, for example, two photons when one photon is off mass shell. The q^2 dependence in this case is that of the pseudoscalar meson pole involved. Here both the currents are conserved. In our case the axial vector current to which π -meson is coupled is partially conserved and this is reflected in the constant f_B/f_π in the valence quark contributions given in Eqs. (24) and (25). But this is exactly cancelled by the equal time commutator contribution so that q^2 dependence of the continuum contribution is given by B -meson pole, which is indistinguishable from the usual vector B^* -meson pole contribution to $f_\pm(q^2)$ in the heavy quark symmetry limit ($M_{B^*} \simeq M_B$).

The effect of continuum contribution as defined above seems to change the parameter λ in B^* contribution to an effective one $\lambda_{eff} = \lambda + \lambda_c/2$ so that

$$f_+(q^2) = \frac{f_B}{2f_\pi} [\lambda_c + 2\lambda] \frac{1}{1 - q^2/M_B^2} \quad (33)$$

$$f_-(q^2) = -\frac{f_B}{2f_\pi} [\lambda_c + 2\lambda] \left\{ 1 + \frac{1}{1 - q^2/M_B^2} \right\} \quad (34)$$

These formulae, as already noted, hold in the chiral limit i.e. for q^2 in the range determined by $E_\pi = (m_B^2 + m_\pi^2 - q^2) / (2m_B) \leq 1\text{GeV}$ or for $q^2 \geq 17\text{ GeV}^2$.

V. CHIRAL SYMMETRY CONSTRAINED MODEL FOR $F_+(Q^2)$ IN THE ENTIRE MOMENTUM TRANSFER

In order to impliment the constraint given in Eq.(33), we use the extrapolation function [28]

$$f_+(q^2) = \frac{f_+(0)}{1 - a q^2/M_B^2 + b (q^2/M_B^2)^2} \quad (35)$$

which involves three parameters $f_+(0)$, a and b . For $E_\pi \simeq m_\pi \rightarrow 0$, or $q^2 \simeq M_B^2$, Eq.(35) should reduce to Eq.(33) which gives

$$1 - a + b = 0 \quad (36)$$

$$\begin{aligned} f_+(0) &= \frac{f_B}{2f_\pi} (\lambda_c + 2\lambda) (a - 2b) \\ &= \frac{f_B}{2f_\pi} (\lambda_c + 2\lambda) (1 - b) \end{aligned} \quad (37)$$

Thus the pole at $q^2 = M_B^2$ is factored out in Eq.(35) and we obtain

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/M_B^2)(1 - b q^2/M_B^2)} \quad (38)$$

It is interesting to note that Eq.(38) implies that in the heavy mass and large E_π ($\gg 1\text{ GeV}$ or $q^2 \ll 17\text{ GeV}^2$) limit, f_+ behaves like $1/E^2$ in agreement with that found earlier in the HQET-LEET (large energy effective theory) formalism for heavy-to-light form factors [29]. The Eq.(38) suggests that we may interpret the second factor in Eq.(38) as arising from a second pole at $q^2 = M_{B'}^2$, where $M_{B'}$ is some effective mass, so that

$$b = \frac{M_B^2}{M_{B'}^2} \quad (39)$$

and

$$f_+(0) = \frac{f_B}{2f_\pi} [\lambda_c + 2\lambda] \left(1 - \frac{M_B^2}{M_{B'}^2} \right) \quad (40)$$

It is tempting to interpret that the suppression factor $(1 - M_B^2/M_{B'}^2)$ in Eq.(40) and the second pole in Eq.(38) as arising from radial excitations of B^* . Then making use of the formula of Ref. [30], we find $M_{B'}/M_B \simeq 1.14$ so that $b \simeq 0.77$. To proceed further so as to obtain numerical estimates we make the following choice of other parameters: $M_{B^*} \approx M_B \simeq 5.28\text{ GeV}$, $m_q/m_b = 0.063$ and $m_b = 4.757\text{ GeV}$ [31], which give $\lambda_c \simeq 0.826$.

We need also the values for f_B and λ which have considerable uncertainty. We use $f_B = 0.187\text{ GeV}$ and $\lambda = 0.5$ for our numerical predictions. Then from Eq.(37), we obtain $f_+(0) = 0.30$. With the above choice of parameters, we plot the form factor of Eq.(38) in Fig. 2. Also shown for comparison are the B^* pole contribution given in Eq.(30), as well as the predictions obtained respectively from light-cone sum rules [24] and on the light front [23]. In Fig. 3 we give the comparison of our prediction for $f_+(q^2)$ with $f_B = 0.150\text{ GeV}$ and those of [5] and [6] as well as that of a recent calculation [32] where the $B - \pi$ transition form factors are obtained for the whole range of q^2 by using a different method of interpolation between small and large values of q^2 . This figure also gives a comparison to

lattice QCD data [33]. In Fig. 4, we plot the pion momentum distribution in units of $|V_{ub}|^2$ while Fig. 5 gives the comparison of our prediction for this distribution with $f_B = 0.150$ GeV and those in references [5] and [6]. In each case we also give this distribution for the B^* pole for comparison which shows that B^* pole is a good approximation to the full form factors upto E_π of ~ 1 GeV.

Finally we calculate the branching ratio for $B \rightarrow \pi l \nu$ using the form factors given in Eq.(38) and $f_+(0)$ in Eq.(40) with our choice of parameters, given previously. Our prediction is $\Gamma = 13.7 |V_{ub}|^2 \text{ ps}^{-1}$ so that using $\tau_B = 1.56 \text{ ps}$ we obtain for the branching ratio

$$\mathcal{B}(B^0 \rightarrow \pi^- l^+ \nu) \simeq 21.4 |V_{ub}|^2 \quad (41)$$

To indicate sensitivity of this result on values of λ and f_B which we take $0.3 \leq \lambda \leq 0.7$ and $0.150 < f_B < 0.187$ GeV, we can express our prediction as

$$\mathcal{B}(B^0 \rightarrow \pi^- l^+ \nu) \simeq (20.0 \pm 11.5) |V_{ub}|^2 \quad (42)$$

With the CLEO measurement [34], the result given in Eq.(41) means

$$|V_{ub}| = (2.90 \pm 0.48) \times 10^{-3} \quad (43)$$

This is consistent with the result from exclusive decays quoted by the Particle Data Group [35]:

$$|V_{ub}| = (3.3 \pm 1.1) \times 10^{-3}$$

and that obtained from the inclusive decays $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ with $V_{cb} = 0.0395$: $|V_{ub}| = (3.2 \pm 0.8) \times 10^{-3}$.

VI. CONCLUSION

We have presented a framework in which constraints from chiral symmetry and quark model are used to predict the form factors for the exclusive $B \rightarrow \pi^+ l^- \nu$ decay in the entire physical range of momentum transfer. The valence quark contribution is calculated in the chiral symmetry approach. This together with the equal time commutator contribution simulate a B-meson pole q^2 -dependence of the form factors. This and B^* pole contribution being obtained in chiral symmetry, are valid for $E_\pi \leq 1$ GeV. This constraint is then implemented through the extrapolation function for $f_+(q^2)$. The resulting form factor, valid for the entire physical range for q^2 , represents a softening of B^* pole behavior and the suppression of the chiral coupling which can be interpreted as being provided by the radial excitation of B .

The shape of the pion momentum distribution shown in Figs. 4 and 5 should be able to distinguish our model from the others. The predicted branching ratio for $B \rightarrow \pi l \nu$ in unit of $|V_{ub}|^2$ is sensitive to the values of λ and f_B as exemplified in Eq.(42), once we select the b quark mass and the ratio m_q/m_b . We emphasize that uncertainties in the predicted branching ratio for $B \rightarrow \pi l \nu$ are due to uncertainties in the external parameters λ and f_B and are not intrinsic to the model.

With the CLEO measurement of the branching ratio for $B \rightarrow \pi l \nu$, our prediction (41) with $\lambda = 0.5$ and $f_B = 0.187$ GeV give $|V_{ub}|$ as in Eq.(43) which is consistent with the value quoted by the Particle Data Group. With some better knowledge of λ , f_B and m_b , the errors in form factors and the branching ratio can potentially be reduced.

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- [1] L. Wolfenstein, Phys. Lett. **B291** (1992) 177
 - [2] G. Burdman and J. F. Donoghue, Phys. Lett. **B280** 1992 287

- [3] R. Casalluoni, A. Deandrea, N. DiBartolomeo, R.Gatto, F.Feruglio and G.Nardulli, Phys. Lett. **B299** (1993) 139
- [4] N. Paver and Riazuddin, Phys. Lett. **B320** (1994) 364
- [5] G. Burdman and J. Kambor, Phys. Rev. **D55** (1997) 2817
- [6] M. Wirbel, S. Stech and M. Bauer, Z. Phys. **C29** (1985) 637; M. Bauer and M. Wirbel, *ibid*, **42** (1989) 671
- [7] J.G. Körner and G.A. Schuler, Z.Phys. **C38** (1988) 511
- [8] C.A. Dominguez and N. Paver, Z. Phys. **C41** (1988) 217
- [9] N. Isgur and D. Scora, Phys. Rev. **D52** (1995) 2783; N. Isgur, D. Scora, B. Grinstein and M. Wise, Phys. Rev. **D39** (1989) 799
- [10] A.A. Ovchinnikov, Phys. Lett. **B229** (1989) 127
- [11] V.L.Chernyak and I.R. Zhitnitsky, Nucl. Phys. **B345** (1990) 137; A.A. Ovchinnikov, Sov. J. Nucl. Phys. **50** (1989) 519
- [12] P. Ball, V.M. Braun, H.G. Dosch, Phys. Lett. **B273** (1991) 316; Phys. Rev. **D44** (1991) 3567; P. Ball, Phys. Rev. **D48** (1993) 3190
- [13] S. Narison, Phys. Lett. **B283** (1992) 283
- [14] M.B. Wise, Phys. Rev. **D45** (1992) 2188
- [15] V.M. Belyaev, A. Khodjamirian and R. Ruckl, Z. Phys. **C60** (1993) 349
- [16] P. Colangelo and P. Santorelli, Phys. Lett. **B327** (1994) 123
- [17] A. Ali, V.M. Braun and H.G. Dosch, Z. Phys. **C63** (1994) 437
- [18] H.Y. Cheng, Chin. J. Phys. **32**, (1994) 425
- [19] G. Burdman, Z. Ligeti, M. Neubert and Y. Nir, Phys. Rev. **D49** (1994) 2331
- [20] V.M. Belyaev et. al., Phys. Rev. **D51** (1995) 617
- [21] H.N. Li and H.L. Yu, Phys. Rev. Lett. **74** (1995) 4388
- [22] D. Melikhov, Phys. Rev. **D53** (1996) 2460
- [23] Chi-Yee Cheung, Chien-Wen Hwang and Wei-Min Zhang, Z. Phys. **C75** (1997) 657
- [24] A. Khodjamirian et al, Phys. Lett. **B410** (1997) 275
- [25] E. Bajan, P. Ball and V.M. Braun, Phys. Lett. **B417** (1998) 154
- [26] A. Khodjamirian and R. Rückl, in Heavy Flavors II, edited by A.J. Buras and M. Linder, World Scientific, Singapore (1998) p.345; A. Khodjamirian, R. Rückl and C.W. Winhart, Phys. Rev. **D58** (1998) 054013
- [27] N. Isgur, Phys. Rev. **D 13**, (1976) 129; **23** (1981) 817(E)
- [28] Report of the BaBar Physics Workshop
- [29] J.Charles ,et al,Phys. Rev.D60 (1999) 014001
- [30] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. **D21** (1980) 23
- [31] see for example, E. Jenkins, Phys. Rev. **D54** (1996) 4515
- [32] Th. Feldmann and P. Kroll, Eur. Phys. J.C12 (2000) 99, hep-ph/9905343; M. Dahm, R. Jakob and P. Kroll, Z. Phys. **C68** (1995) 595
- [33] J.M. Flynn, Proc. of 28th. Int. Conference on High Energy Physics, Warsaw(1996),hep-lat/9610016
- [34] CLEO Collaboration, J.P. Alexander, et al, Phys. Rev. Lett. **77** (1996) 5000
- [35] Particle Data Group, Euro.J.Phys. **C3** (1998) 104

VIII. FIGURE CAPTIONS

Figure 1 Valence-quark contribution.

Figure 2 The form factor $f_+(q^2)$ as a function of the momentum transfer $t = q^2$. The solid curve is our prediction for $\lambda = 0.5$ and $f_B = 0.187$ GeV. The dashed line is the B^* -pole contribution as given in Eq. (30). The dotted and dashed-dotted lines are respectively the predictions of Refs. [23] and [24].

Figure 3 Same as those in Figure 2 but for $f_B = 0.150$ GeV. The dashed-dotted, dotted and dash-dot-dot lines are respectively the predictions of Refs. [5], [6] and [32]. The data points correspond to lattice results [33].

Figure 4 The pion energy distribution in units of $|V_{ub}|^2$ as function of pion energy. The dashed line is B^* prediction.

Figure 5 Same as in Fig. 4 for $f_B = 0.150$ GeV. The dashed-dotted and dotted lines are respectively the predictions of Ref. [5] and [6].

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